## Price indeterminacy in day-ahead market

## Mid-Price rule

A "price indeterminacy" is a situation in which at least two feasible solutions with the same matched volume, the same block and MIC selections and the same welfare, exist but which only differ by their market price.

In this case, all these solutions have an identical welfare, and it is necessary to define a rule to choose between the set of valid prices.

In case of non-unique price solution, for a given hour $h$, the mid-point rule will apply, defined as the minimal square distance between $\mathrm{MCP}_{\mathrm{m}, \mathrm{h}}$ and the distance to its "midpoint". In case of several indeterminacies, the sum of these distances is considered. The mid-point price expression can be formulated with the following function:

$$
\min \sum_{m, h}\left(M C P_{m, h}-\frac{U B_{m, h}+L B_{m, h}}{2}\right)^{2}
$$

Being:

- $\mathrm{LB}_{\mathrm{m}, \mathrm{h}}$ : the lowest possible price in the market m on hour h for a defined net position Q*. $^{*}$.
- $U_{m, h}$ : the highest possible price in the market $m$ on hour $h$ for a defined net position $Q^{*}$.
- $\mathrm{MCP}_{\mathrm{m}, \mathrm{h}}$ : the (unrounded) Market Clearing Price of the market $m$ on hour $h$.
- The mid-point of the intersection defined by $\left[\mathrm{LB}_{\mathrm{m}, \mathrm{h}}, \mathrm{UB} \mathrm{m}_{\mathrm{m}, \mathrm{h}}\right]$ is equal to $\left(U B_{m, h}+L B_{m, h}\right) / 2$.

Subject to the following constraints:

- All market and network constraints
- NO_PAB constraints
- PUN imbalance constraints if applicable
- B_ACCEPTo = 1 if the block order o is executed in the feasible solution $=0$ otherwise
- B_ACCEPTMIC $=1$ if the complex order MIC is executed in the feasible solution

$$
=0 \text { otherwise }
$$

NOTE: The following rule is applied only in the case on which a lower bound and an upper bound exist for every bidding area involved in indeterminacy problem

## Mid price rule in one isolated bidding area

Given a certain bidding area 1, let's consider the following situation:


Every price between the interval [50,01;49,70] returns a feasible solution with the same welfare.

By applying mid-price rule:

$$
\min \sum\left(m c p_{m . h}-\frac{U B_{m, h+} L B_{m, h}}{2}\right)^{2}
$$

Marginal clearing price for bidding area 1 is $49,855 € / \mathrm{MWh}$

## Case 1:

| SESSION_ID | PERIOD | ORDER_ID | TYPE | BIDDINGAREA_ID | MO | QO | PO |
| ---: | ---: | ---: | :--- | ---: | ---: | ---: | ---: |
| 1 | 1 | 10501 | SUPPLY | 1 | 2 | 14 | 10 |
| 1 | 1 | 10500 | DEMAND | 1 | 1 | 15 | 60 |
| 1 | 1 | 10503 | SUPPLY | 2 | 4 | 6 |  |
| 1 | 1 | 10504 | SUPPLY | 2 | 5 | 5 | 58 |
| 1 | 1 | 10502 | DEMAND | 2 | 3 | 5 | 50 |

Let's consider 2 bidding areas (1 and 2), connected with unlimited ATC.
As shown in the graph of which below, a price indeterminacy exist in the interval [30,00; 50,00 ]


By Applying mid-price rule:

$$
\left(\min \left(x-\frac{10,00+60,00}{2}\right)^{2}+\left(x-\frac{30,00+50,00}{2}\right)^{2}\right)
$$

The clearing price is:
$4 \mathrm{X}=150 \rightarrow \mathrm{X}=37,5 € / \mathrm{MWh}$

## Case 2:

| SESSION_ID | PERIOD | ORDER_ID | TYPE | BIDDINGAREA_ID | MO | QO | PO |
| ---: | ---: | ---: | :--- | ---: | ---: | ---: | ---: |
| 1 | 1 | 10500 | DEMAND | 1 | 1 | 1 | 115 |
| 1 | 1 | 10501 | SUPPLY | 1 | 2 | 50 |  |
| 1 | 1 | 10502 | DEMAND | 2 | 3 | 5 | 410 |
| 1 | 1 | 10503 | SUPPLY | 2 | 4 | 400 |  |
| 1 | 1 | 10504 | SUPPLY | 2 | 5 | 30 |  |

Let's consider 2 bidding areas (1 and 2), connected with unlimited ATC.
As shown in the graph of which below, a price indeterminacy exist in the interval [30,00; 55,00 ]


By Applying mid-price rule:

$$
\left(\min \left(x-\frac{55+110}{2}\right)^{2}+\left(x-\frac{30+400}{2}\right)^{2}\right)
$$

From which:
$4 \mathrm{X}=595 \rightarrow \mathrm{X}=148,75 € / \mathrm{MWh}$
Since $148,75>55$ (upper bound of clearing price interval), price is lowered until upper bound of clearing price interval: $55 € / \mathrm{MWh}$

## Price indeterminacy - An exception

Above described rule is not applied when, in the bidding area where price indeterminacy happens, it's not present an upper bound or a lower bound.

For example:


As shown in the picture, for bidding area 1:

- There's an upper bound ( $400 € / \mathrm{MWh}$ )
- There is no lower bound

In a case like this:

1. the algorithm considers the interval between minimum theoretical price and maximum one: $[-500 ;+400]^{1}$;
2. Since bidding area 1 imports 100 MWh from bidding area 2 , minimum acceptable price cannot be -500 , but it should be equal to bidding area 2 clearing price ( 27 €/MWh);
3. the algorithm chooses between minimum theoretical price ( $-500 € / \mathrm{MWh}$ ) and maximum one ( $400 € / \mathrm{MWh}$ ) randomly;
4. In the case on which CPLEX chooses maximum price ( $400 € / \mathrm{MWh}$ ), solution is valid;
5. In the case on which CPLEX chooses minimum theoretical price ( $-500 € / \mathrm{MWh}$ ), solution is not valid ( $-500<27$ ). Consequently CPLEX starts a series of iteration to set bidding area 1 price equal to bidding area 2 price ( $27 € / \mathrm{MWh}$ ).

Similarly, given two bidding areas and a set of OBKs as the following one:

[^0]| SESSION_ID | PERIOD | ORDER_ID | MO | BIDDINGAREA_ID | TYPE | QO | PO |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 1 | 10501 | 2 | 1 | SUPPLY | 60 | 0,01 |
| 8 | 1 | 10502 | 3 | 1 | SUPPLY | 60 | 50 |
| 8 | 1 | 10500 | 1 | 2 | DEMAND | 115 | 400 |
| 8 | 1 | 10506 | 10 | 2 | DEMAND | 5 | 55 |

In this case:

- Upper bound in bidding area 1 is missing
- Lower bound in bidding area 2 is missing

Mid price rule is not applied and price is set randomly equal to 55 or 50 .


[^0]:    ${ }^{1}$ Note: Even if minimum offer price for GME's bidding area is $0 € / \mathrm{MWh}$, minimum clearing price could is set to -500 €/MWh

